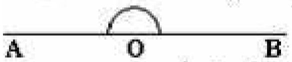
GEOMETRIE PLANĂ ŞI ÎN SPAŢIU

Segmentele congruente au măsurile egale:
$[AB] \equiv [CD] \Leftrightarrow AB = CD$.
Mijlocul unui segment: $M \in [AB]$ şi $[MA] \equiv [MB]$.

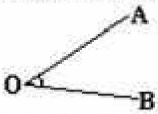
Unghiuri

Unghi alungit: $m(\hat{AOB}) = 180^\circ$

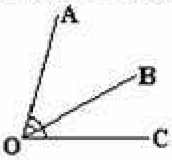


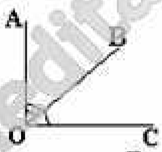
Unghi nul: $m(\hat{AOB}) = 0^\circ$

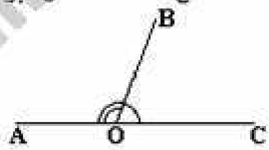
Unghi propriu: $m(\hat{AOB}) = n^\circ \in (0^\circ, 180^\circ)$

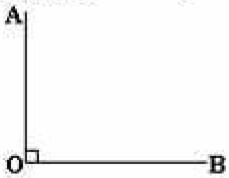


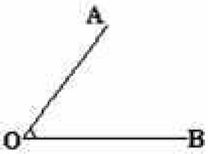
Unghiuri congruente: $\hat{AOB} \equiv \hat{A'O'B'} \Leftrightarrow m(\hat{AOB}) = m(\hat{A'O'B'})$

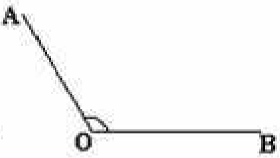
Unghiuri adiacente:


Unghiuri adiacente complementare:


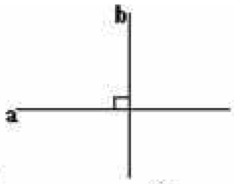
Unghiuri adiacente suplementare:


Unghi drept: $m(\hat{AOB}) = 90^\circ$


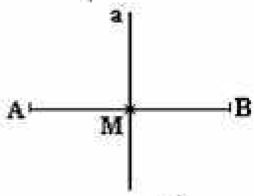
Unghi ascuţit: $m(\hat{AOB}) < 90^\circ$


Unghi obtuz: $m(\hat{AOB}) > 90^\circ$; $m(\hat{AOB}) < 90^\circ$


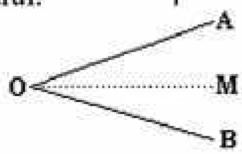
Unghiuri opuse la vârf sunt congruente: $\hat{AOB} \equiv \hat{COD}$ şi $\hat{BOD} \equiv \hat{AOC}$



Drepte perpendiculare:

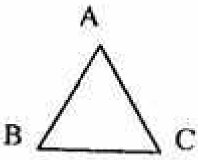


Mediatoarea segmentului:

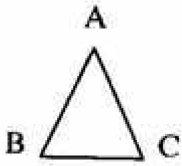


Bisectoarea unghiului:

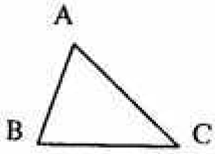
Triunghiul



- ***echilateral***: $AB = AC = BC = a$

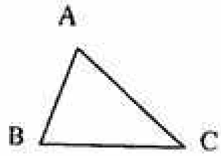


- ***isoscel***: $[AB] \equiv [AC]$

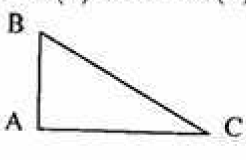


- ***scalen*** (oarecare): $AB \neq AC$; $AB \neq BC$; $AC \neq BC$

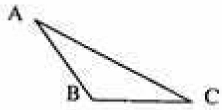
Clasificarea triunghiurilor după unghiuri:



- ***ascuțitunghic***: $m(\hat{A}) < 90^\circ$; $m(\hat{B}) < 90^\circ$; $m(\hat{C}) < 90^\circ$

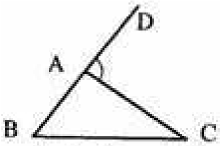


- ***dreptunghic***: $m(\hat{A}) = 90^\circ$



- ***obtuzunghic***: $m(\hat{B}) > 90^\circ$

Unghi exterior



$m(D\hat{A}C) = m(\hat{B}) + m(\hat{C})$

$m(\hat{A}) + m(\hat{B}) + m(\hat{C}) = 180^\circ$

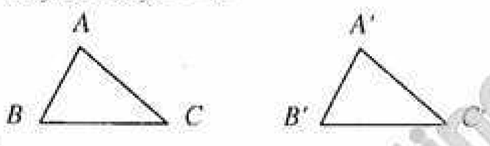
$a > b \Leftrightarrow m(\hat{A}) > m(\hat{B})$

Perimetrul triunghiului: $P = 2p = a + b + c$ **sau** $2p = AB + BC + CA$

$AB = AC \Leftrightarrow \hat{B} \equiv \hat{C}$

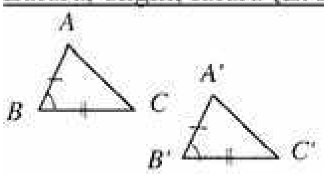
Congruența triunghiurilor:

$$\Delta ABC \equiv \Delta A'B'C' \Leftrightarrow \begin{cases} [AB] \equiv [A'B'], [AC] \equiv [A'C'], [BC] \equiv [B'C'] \\ \hat{A} \equiv \hat{A}', \hat{B} \equiv \hat{B}', \hat{C} \equiv \hat{C}' \end{cases}$$



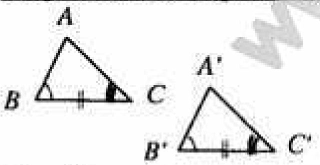
Cazurile de congruență:

Latură, unghi, latură (L.U.L.):



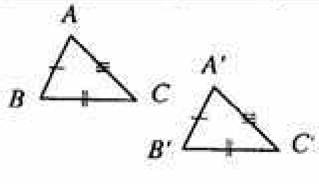
$$\left. \begin{array}{l} [AB] \equiv [A'B'] \\ \hat{B} \equiv \hat{B}' \\ [BC] \equiv [B'C'] \end{array} \right\} \Rightarrow \Delta ABC \equiv \Delta A'B'C'$$

Unghi, latură, unghi (U.L.U.):



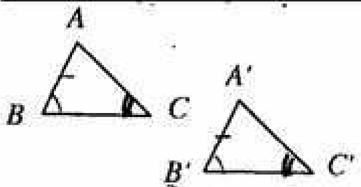
$$\left. \begin{array}{l} \hat{B} \equiv \hat{B}' \\ [BC] \equiv [B'C'] \\ \hat{C} \equiv \hat{C}' \end{array} \right\} \Rightarrow \Delta ABC \equiv \Delta A'B'C'$$

Latură, latură, latură (L.L.L.):



$$\left.\begin{array}{l}[AB]\equiv[A'B'] \\ [AC]\equiv[A'C'] \\ [BC]\equiv[B'C']\end{array}\right\}\Rightarrow \Delta ABC\equiv\Delta A'B'C'$$

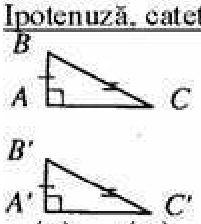
Latură, unghi, unghi (L.U.U.):



$$\left.\begin{array}{l}[AB]\equiv[A'B'] \\ \hat{B}\equiv\hat{B}' \\ \hat{C}\equiv\hat{C}'\end{array}\right\}\Rightarrow \Delta ABC\equiv\Delta A'B'C'$$

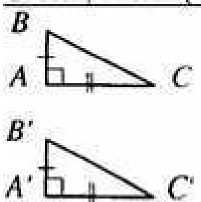
Cazurile de congruență a triunghiurilor dreptunghice

Ipotenuză, catetă (I.C):



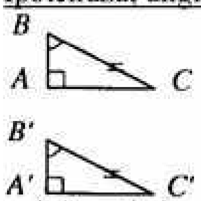
$$\left.\begin{array}{l}m(\hat{A})\equiv m(\hat{A}')=90^\circ \\ [BC]\equiv[B'C'] \\ [AB]\equiv[A'B']\end{array}\right\}\Rightarrow \Delta ABC\equiv\Delta A'B'C'$$

Catetă, catetă (C.C.):



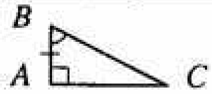
$$\left.\begin{array}{l}m(\hat{A})\equiv m(\hat{A}')=90^\circ \\ [AB]\equiv[A'B'] \\ [AC]\equiv[A'C']\end{array}\right\}\Rightarrow \Delta ABC\equiv\Delta A'B'C'$$

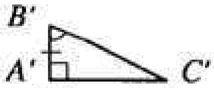
Ipotenuză, unghi ascuțit (I.U.):



$$\left.\begin{array}{l}m(\hat{A})\equiv m(\hat{A}')=90^\circ \\ [BC]\equiv[B'C'] \\ \hat{B}\equiv\hat{B}'\end{array}\right\}\Rightarrow \Delta ABC\equiv\Delta A'B'C'$$

Catetă, unghi ascuțit (C.U.):

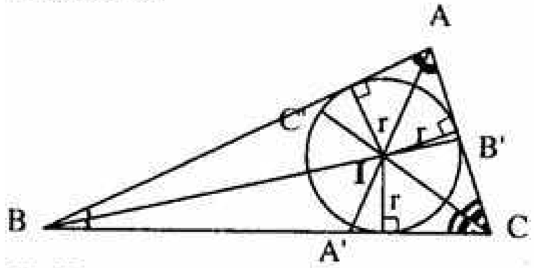




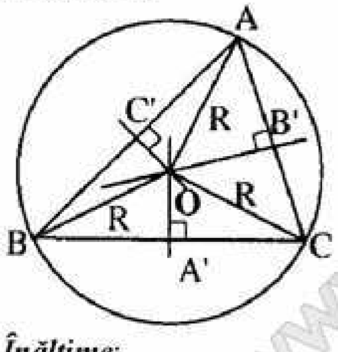
$$\left.\begin{array}{l} m(\hat{A}) \equiv m(\hat{A}') = 90^\circ \\ [AB] \equiv [A'B'] \\ \hat{B} \equiv \hat{B}' \end{array}\right\} \Rightarrow \Delta ABC \equiv \Delta A'B'C'$$

Linii importante în triunghi:

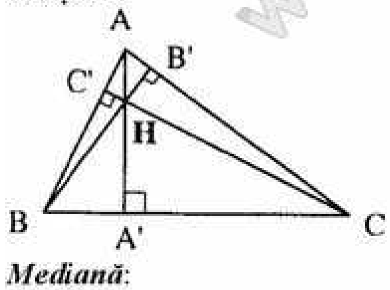
Bisectoare:



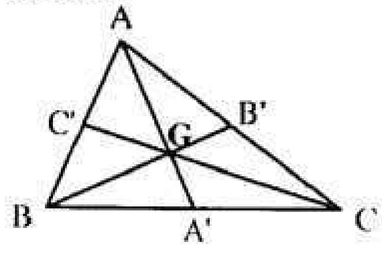
Mediatoare:



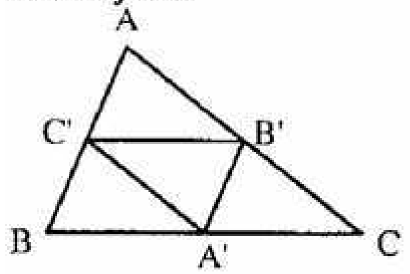
Înălțime:



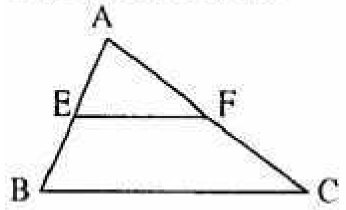
Mediană:



***Linia mijlocie*:**



Teorema lui Thales:



$$\left.\begin{matrix} EF \parallel BC \\ E \in AB \\ F \in AC \end{matrix}\right\} \Rightarrow \frac{AE}{EB} = \frac{AF}{FC} \Leftrightarrow \frac{BE}{AB} = \frac{CF}{AC} \Leftrightarrow \frac{BE}{AE} = \frac{CF}{AF}$$

Teorema reciprocă a teoremei lui Thales

$$\left.\begin{matrix} E \in AB \\ F \in AC \\ \frac{AE}{EB} = \frac{AF}{FC} \end{matrix}\right\} \Rightarrow EF \parallel BC$$

Asemănarea triunghiurilor

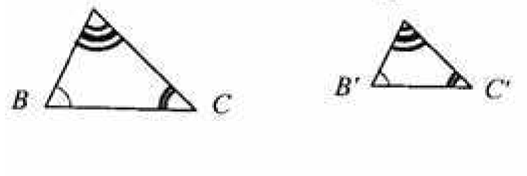
$$\Delta ABC \sim \Delta A'B'C' \Leftrightarrow \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'} \text{ şi } \hat{A} \equiv \hat{A}'; \quad \hat{B} \equiv \hat{B}'; \quad \hat{C} \equiv \hat{C}'$$

Cazuri de asemănare:

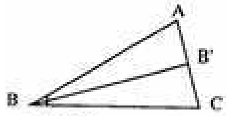
1. $\left.\begin{matrix} \hat{A} \equiv \hat{A}' \\ \hat{B} \equiv \hat{B}' \end{matrix}\right\} \Rightarrow \Delta ABC \sim \Delta A'B'C'$

2. $\left.\begin{matrix} \hat{A} \equiv \hat{A}' \\ \frac{AB}{A'B'} = \frac{AC}{A'C'} \end{matrix}\right\} \Rightarrow \Delta ABC \sim \Delta A'B'C'$

3. $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'} \Rightarrow \Delta ABC \sim \Delta A'B'C'$

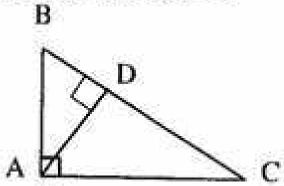


Teorema bisectoarei:



$$\left.\begin{array}{l}\Delta ABC \\ B' \in AC \\ A\hat{B}B' \equiv B'\hat{B}C\end{array}\right\} \Rightarrow \frac{B'A}{B'C} = \frac{BA}{BC}$$

Teorema catetei:

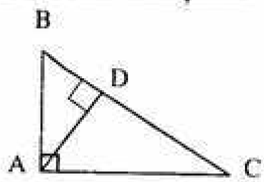


$m(\hat{A}) = 90^\circ$

$AD \perp BC \Rightarrow AB^2 = BC \cdot BD$

$AC^2 = BC \cdot DC$

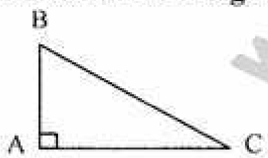
Teorema înălțimii:



$m(\hat{A}) = 90^\circ$

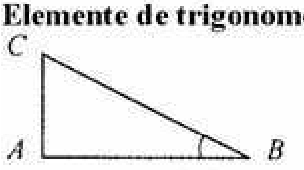
$AD \perp BC \Rightarrow AD^2 = DB \cdot DC \Rightarrow AD = \sqrt{DB \cdot DC}$

Teorema lui Pitagora:



$m(\hat{A}) = 90^\circ \Rightarrow BC^2 = AB^2 + AC^2$ sau pentru catete: $AB^2 = BC^2 - AC^2$; $AC^2 = BC^2 - AB^2$

Elemente de trigonometrie



$\sin(\sphericalangle \alpha) = \frac{cateta\ opusa}{ipotenuza}$; $\sin(\sphericalangle B) = \frac{AC}{BC}$; $\sin(\sphericalangle C) = \frac{AB}{BC}$

$\cos(\prec\alpha)=\dfrac{cateta\ alaturata}{ipotenuza}$; $\cos(\prec B)=\dfrac{AB}{BC}$; $\cos(\prec C)=\dfrac{AC}{BC}$

$tg(\prec\alpha)=\dfrac{cateta\ opusa}{cateta\ alaturata}$; $tg(\prec B)=\dfrac{AC}{AB}$; $tg(\prec C)=\dfrac{AB}{AC}$

$ctg(\prec\alpha)=\dfrac{cateta\ alaturata}{cateta\ opusa}$; $tg(\prec B)=\dfrac{AC}{AB}$; $ctg(\prec C)=\dfrac{AC}{AB}$

$\sin(\prec\alpha)=\cos(\prec(90^\circ-\alpha))$

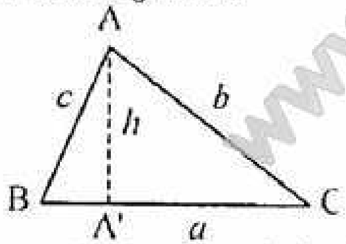
$\cos(\prec\alpha)=\sin(\prec(90^\circ-\alpha))$

$tg(\prec\alpha)=\dfrac{\sin(\prec\alpha)}{\cos(\prec\alpha)}$; $tg(\prec\alpha)=\dfrac{1}{ctg(\prec\alpha)}$

$ctg(\prec\alpha)=\dfrac{\cos(\prec\alpha)}{\sin(\prec\alpha)}$; $ctg(\prec\alpha)=\dfrac{1}{tg(\prec\alpha)}$

Unghiul	sin	cos	tg	ctg
0°	0	1	0	-
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90°	1	0	-	0

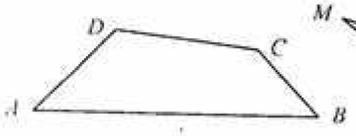
Aria triunghiului:

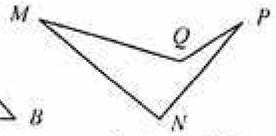


$S=\dfrac{BC\cdot h}{2}$, $S=\dfrac{a\cdot b\cdot\sin C}{2}$

$S=\sqrt{p(p-a)(p-b)(p-c)}$, unde $p=\dfrac{a+b+c}{2}$.

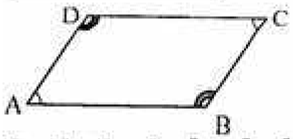
Patrulatere

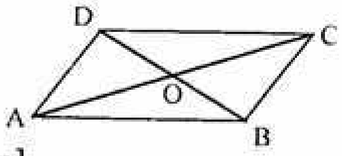




Suma măsurilor unghiurilor $m(\hat{A})+m(\hat{B})+m(\hat{C})+m(\hat{D})=360^{o}$

Paralelogramul:





$[AB]\equiv[CD]$; $[AD]\equiv[BC]$

$\hat{A}\equiv\hat{C}$; $\hat{B}\equiv\hat{D}$

$m(\hat{A})+m(\hat{B})=180^{o}$

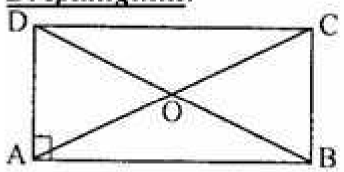
$m(\hat{A})+m(\hat{D})=180^{o}$

$AC\cap BD=\{O\}$ astfel încât $[OA]\equiv[OC]$ şi $[OB]\equiv[OD]$.

Perimetrul paralelogramului: $2p=2AB+2BC$

Aria paralelogramului: $S=B\cdot h=\dfrac{D\cdot d\cdot\sin u}{2}=AB\cdot AD\cdot\sin A$

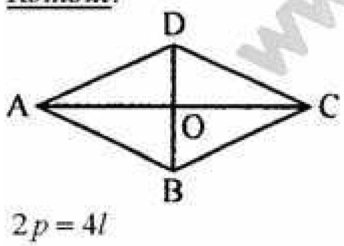
Dreptunghiul:



$2p=2L+2l=2(L+l)$

$S=L\cdot l=\dfrac{d^2\cdot\sin u}{2}$

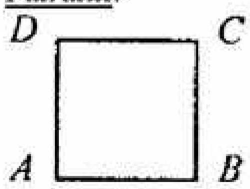
Rombul:



$2p=4l$

$S=\dfrac{D\cdot d}{2}=l^2\cdot\sin A$

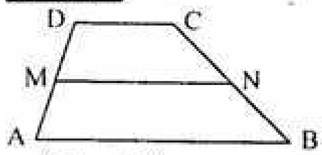
Pătratul:

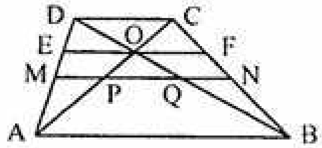


$2p = 4l$

$S = \frac{d^2}{2}$

Trapezul:





$m(\hat{A}) + m(\hat{D}) = 180^\circ$

$m(\hat{B}) + m(\hat{C}) = 180^\circ$

$MN \parallel AB$; $MN \parallel CD$; $MN = \frac{AB + CD}{2}$

$PQ = \frac{AB - CD}{2}$

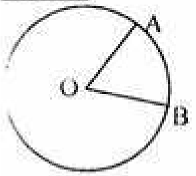
$MP = QN = \frac{CD}{2}$; $EF = \frac{2 \cdot AB \cdot CD}{AB + CD}$

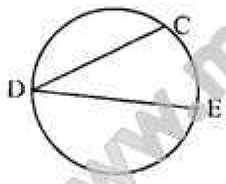
$PQ = MP = NQ \Leftrightarrow AB = 2 \cdot CD$

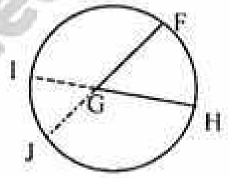
$2p = AB + BC + CD + DA$

$S = \frac{(B + b) \cdot h}{2} = l.m. \cdot h$, unde $l.m.$ = lungimea liniei mijlocii

Cercul:







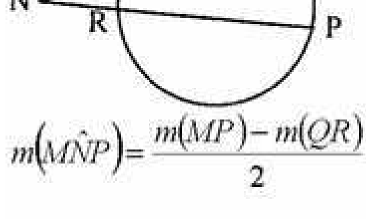
$B \in C(O, R) \Leftrightarrow OB = R$

$C \in Int\ C(O, R) \Leftrightarrow OC < R$

$A \in Ext\ C(O, R) \Leftrightarrow OA > R$

$m(A\hat{O}B) = m(AB)$

$m(C\hat{D}E) = \frac{m(CE)}{2}$



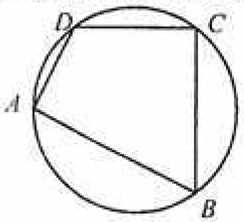
$m(M\hat{N}P) = \frac{m(MP) - m(QR)}{2}$

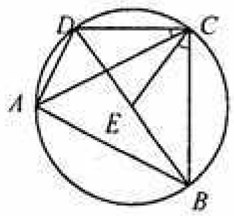
$m(\hat{EGH}) = \frac{m(EH) + m(IJ)}{2}$

$L = 2\pi R$

$S = \pi R^2$

Teorema lui Ptolomeu





Un patrulater ABCD este inscriptibil, dacă şi numai dacă are loc relaţia: $AC \cdot BD = AB \cdot CD + AD \cdot BC$

Aria poligonului regulat cu n laturi înscris în cercul de rază R

$S_n = \frac{1}{2} \cdot a_n \cdot p_n$, $a_n =$ apotema; $p_n =$ perimetrul

Triunghiul echilateral

$L_3 = R\sqrt{3}$, $a_3 = \frac{1}{2}R$, $S_3 = \frac{3}{4}R^2\sqrt{3}$; $S_3 = \frac{l^2\sqrt{3}}{4}$

Pătratul

$L_4 = R\sqrt{2}$, $a_4 = R\frac{\sqrt{2}}{2}$, $S_4 = 2R^2$

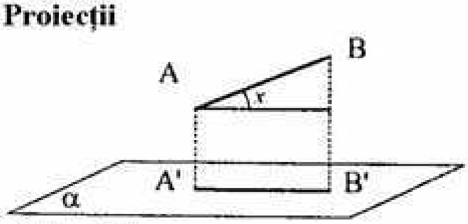
Hexagonul regulat

$L_6 = R$, $a_6 = R\frac{\sqrt{3}}{2}$, $S_6 = \frac{3}{2}R^2\sqrt{3}$

Poligonul regulat cu n laturi

$L_n = 2R\sin\frac{180^\circ}{n^\circ}$, $a_n = R \cdot \cos\frac{180^\circ}{n^\circ}$, $S_n = \frac{n}{2} \cdot R^2 \sin\frac{180^\circ}{n^\circ}$

Proiecţii

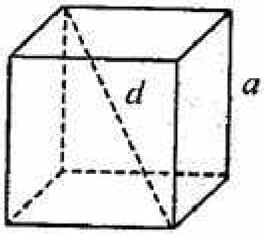


$A'B' = AB \cdot \cos x$

$S_{\Delta A'B'C'} = S_{\Delta ABC} \cdot \cos x$

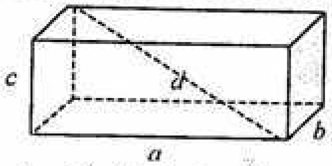
Arii şi volume ($A_l =$ aria laterală; $A_b =$ aria bazei; $A_t =$ aria totală; $h =$ înălţimea; $V =$ volumul; $P =$ perimetrul bazei)

Cubul:



$A_t = 6a^2 = 2d^2$

Paralelipipedul dreptunghic:

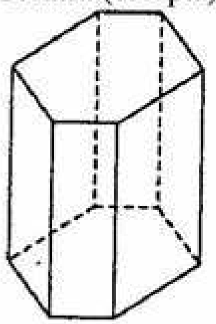


$A_t = 2(ab + bc + ac)$

$V = a \cdot b \cdot c$

$d^2 = a^2 + b^2 + c^2$

Prisma (dreaptă):

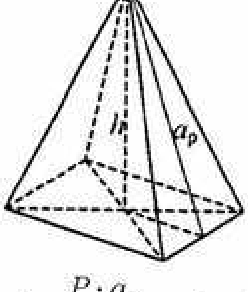


$A_l = P \cdot h$

$A_t = A_l + 2 \cdot A_b$

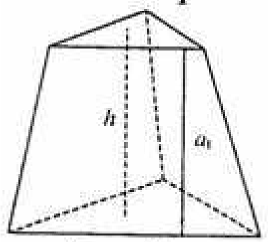
$V = A_b \cdot h$

Piramida regulată



$A_l = \frac{P \cdot a_p}{2}$; $A_t = A_l + A_b$; $V = \frac{A_b \cdot h}{3}$

Trunchiul de piramidă



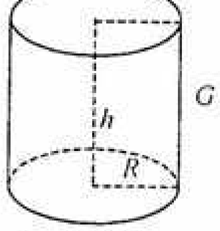
$A_l = \frac{(P+p)\cdot a_t}{2}$

$A_t = A_l + B + b$

$V = \frac{h}{3}\left(B + b + \sqrt{B\cdot b}\right)$

Ariile și volumele corpurilor rotunde (R,r = razele bazelor;G = generatoarea; h = înălțimea)

Cilindrul circular drept



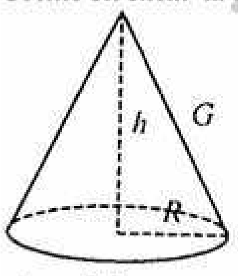
$A_l = 2\pi RG$

$A_t = 2\pi R(G+R)$

$V = \pi R^2 h$

$G = h$

Conul circular drept



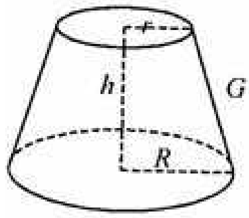
$A_l = \pi RG$

$A_t = \pi R(G+R)$

$V = \frac{1}{3}\left(\pi R^2 h\right)$

$G^2 = R^2 + h^2$

Trunchiul de con circular drept

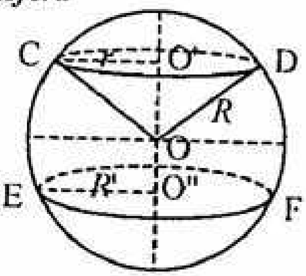


$A_l = \pi G(R+r)$

$A_t = A_l + \pi(R^2 + r^2)$

$V = \frac{\pi h}{3}(R^2 + r^2 + R \cdot r)$

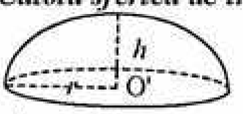
Sfera



$A = 4\pi R^2$

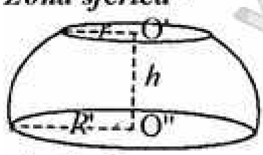
$V = \frac{4\pi}{3} R^3$

Calota sferică de înălțime h*



$A = 2\pi Rh$

Zona sferică *



$A = 2\pi Rh$