

GEOMETRIE

Poliedre

VIII.1. Prisma

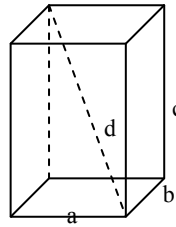
1. Paralelipipedul dreptunghic

$$A_{\text{lat}} = 2(a + b)c;$$

$$A_{\text{tot}} = 2(ab + ac + bc);$$

$$V = abc$$

$$d^2 = a^2 + b^2 + c^2$$



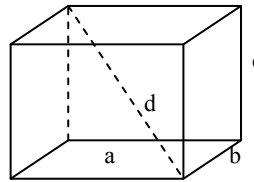
2. Cubul

(de latură $a = b = c$)

$$A = 6a^2$$

$$V = a^3$$

$$a = a\sqrt{3}$$

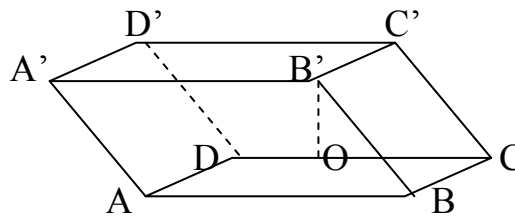


3. Paralelipipedul

$B'O \perp (ABC)$

$B'O = h$

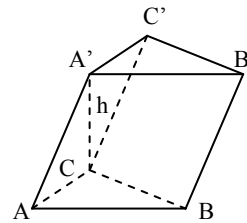
$$V = A_{\text{ABCD}} \cdot h$$



4. Prisma

(dreaptă sau oblică, de înălțime h)

$$V = A_{\text{bazei}} \cdot h$$



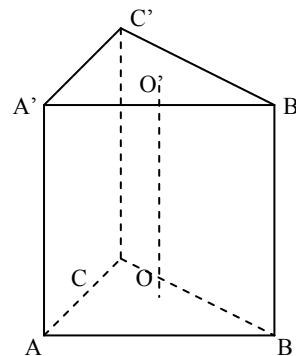
5. Prisma triunghiulară regulată

($AB = a$)

$$A_{\text{lat}} = 3a \cdot h$$

$$A_{\text{tot}} = 3a \cdot h + \frac{a^2 \sqrt{3}}{2}$$

$$V = \frac{a^2 \sqrt{3}}{4} \cdot h$$



VIII.2. Piramida

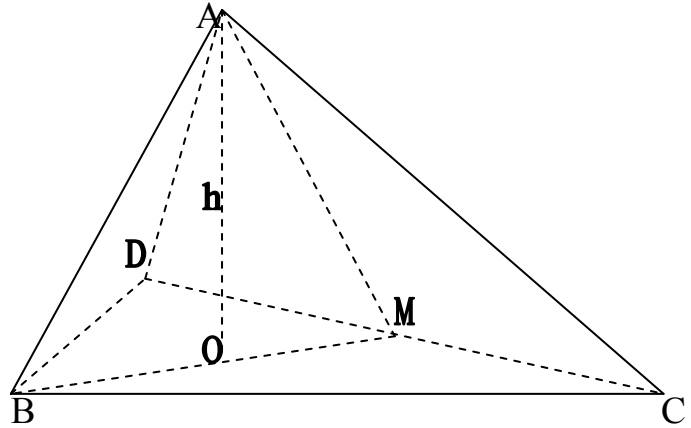
1. Tetraedrul regulat

(toate muchiile sunt congruente,
 $AO \perp (BCD)$, $AM \perp DC$)

$$h = \frac{a\sqrt{6}}{3}, AM = \frac{a\sqrt{3}}{2};$$

$$\sin \hat{A}BO = \frac{\sqrt{6}}{3}, \sin \hat{A}MO = \frac{2\sqrt{2}}{3}$$

$$A = a^2\sqrt{3}; V = \frac{a^3\sqrt{2}}{12}$$



2. Tetraedrul dreptunghic

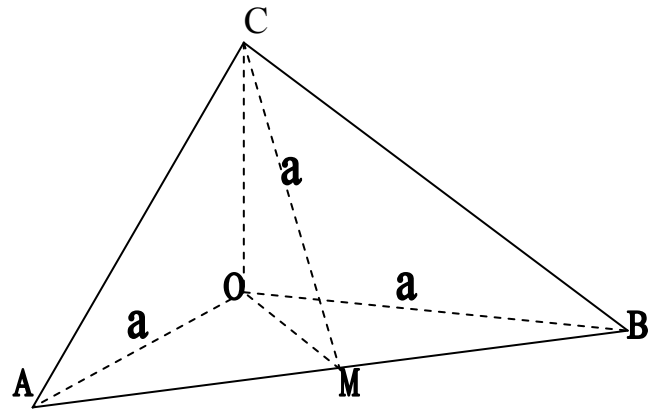
($OA \perp OB \perp OC \perp OA$,
 $OA = OB = OC = a$, $CM \perp AB$)

$$OM = \frac{a\sqrt{2}}{2}, CM = \frac{a\sqrt{6}}{2}; AB = a\sqrt{2}$$

$$A_{ABC} = \frac{a^2\sqrt{3}}{2}$$

$$A_{tot} = \frac{3a^2}{2} + \frac{a^2\sqrt{3}}{2}$$

$$V = \frac{a^3}{6}$$



3. Piramida triunghiulară regulată

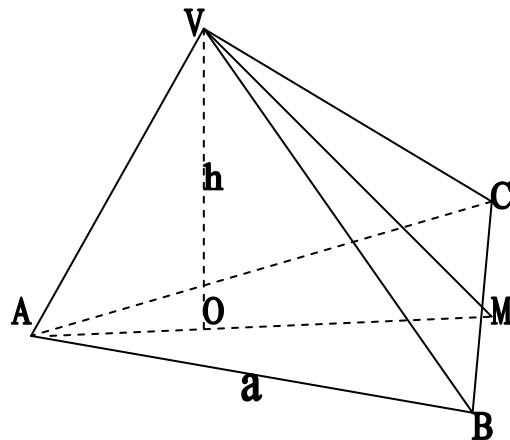
($AB = AC = BC = a$, $VA = VB = VC$
 $VM \perp BC$, VM – apotemă)

$$VM = \sqrt{h^2 + \frac{a^2}{12}}$$

$$A_{lat} = \frac{3a \cdot VM}{2}$$

$$A_{tot} = \frac{a^2\sqrt{3}}{4} + \frac{3a \cdot VM}{2}$$

$$V = \frac{a^2\sqrt{3}}{4} \cdot \frac{h}{3}$$



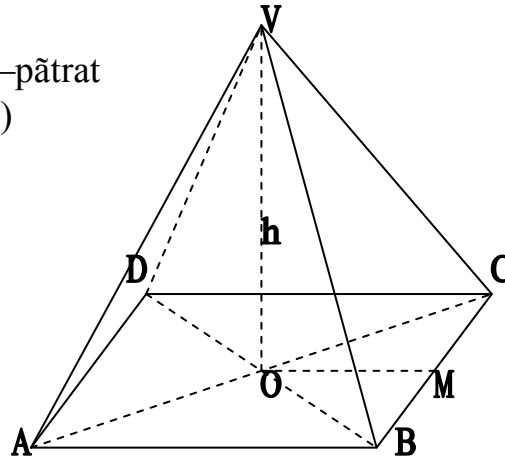
4. Piramida patrulateră regulată (ABCD—pătrat de latură a , $VA = VB = VC = VD$, $VM \perp BC$)

$$VM = \sqrt{h^2 + \frac{a^2}{4}}$$

$$A_{lat} = 2a \cdot VM$$

$$A_{tot} = a^2 + 2a \cdot VM$$

$$V = \frac{a^2 \cdot h}{3}$$



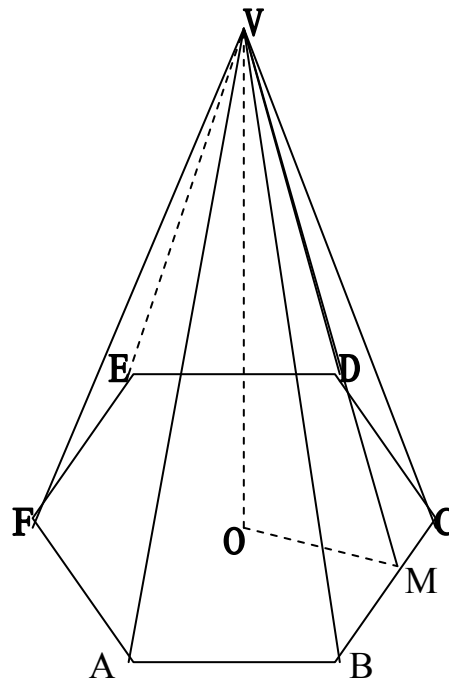
5. Piramida hexagonală regulată (ABCDEF – hexagon regulat $VM \perp BC$, $VA = VB = VC = VD = VE = VF = a$)

$$VM = \sqrt{h^2 + \frac{3a^2}{4}}$$

$$A_{lat} = 3a \cdot VM$$

$$A_{tot} = \frac{3a^2 \sqrt{3}}{2} + 3a \cdot VM$$

$$V = \frac{a^2 \sqrt{3} h}{2}$$



6. Piramida regulată

(piciorul înălțimii coincide cu centrul circumscris bazei):

$$A_{lat} = \frac{P_{bazei} \cdot apotema}{2}$$

$$A_{tot} = A_{bazei} + A_{lat}; V = \frac{A_{bazei} \cdot h}{3}$$

7. Piramida (de înălțime h):

$$A_{tot} = A_{bazei} + A_{lat}; V = \frac{A_{bazei} \cdot h}{3}$$

VIII.3. Trunchiul de piramidă

(B – aria bazei mari, b – aria bazei mici, h – înălțimea)

1. Trunchiul de piramidă oarecare:

$$V = \frac{h}{3}(B + b + \sqrt{B \cdot b})$$

2. Trunchiul de piramidă regulat

P – perimetrul bazei mari,

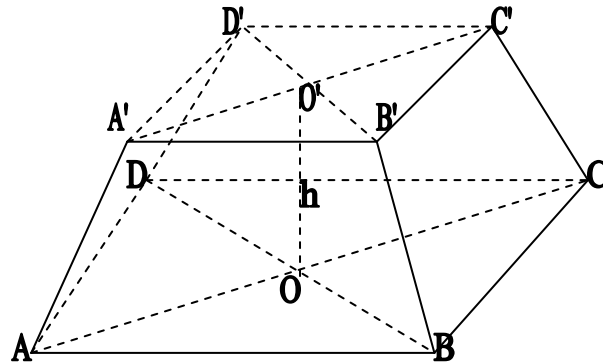
p – perimetrul bazei mici,

a_p – apotema

$$A_{lat} = \frac{(P + p)a_p}{2}$$

$$A_{tot} = B + b + \frac{(P + p)a_p}{2}$$

$$V = \frac{h}{3}(B + b + \sqrt{B \cdot b})$$



VIII.4. Poliedrul regulat

Relația lui Euler: $v - m + f = 2$

(v – numărul vârfurilor, m – numărul muchiiilor, f – numărul fețelor)

Tipurile de poliedre regulate:

- tetraedrul regulat: $f = 4, v = 4, m = 6$;
- cubul (hexaedru regulat): $f = 6, v = 8, m = 12$;
- octaedrul regulat: $f = 8, v = 6, m = 12$;
- dodecaedrul regulat: $f = 12, v = 20, m = 30$;
- icosaedrul regulat: $f = 20, v = 12, m = 30$;